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Fast Color Quantization Using Weighted Sort-Means Clustering

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Color quantization is an important operation with numerous applications in graphics and image processing. Most quantization methods are essentially based on data clustering algorithms. However, despite its popularity as a general purpose clustering algorithm, k-means has not received much respect in the color quantization literature because of its high computational requirements and sensitivity to initialization. In this paper, a fast color quantization method based on k-means is presented. The method involves several modifications to the conventional (batch) k-means algorithm including data reduction, sample weighting, and the use of triangle inequality to speed up the nearest neighbor search. Experiments on a diverse set of images demonstrate that, with the proposed modifications, k-means becomes very competitive with state-of-the-art color quantization methods in terms of both effectiveness and efficiency.

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1. Introduction

True-color images typically contain thousands of colors, which makes their display, storage, transmission, and processing problematic. For this reason, color quantization (reduction) is commonly used as a preprocessing step for various graphics and image processing tasks. In the past, color quantization was a necessity due to the limita-

tions of the display hardware, which could not handle the 16 million possible colors in 24-bit images. Although 24-bit display hardware has become more common, color quantization still maintains its practical value [1]. Modern applications of color quantization include: (i) image compression [2], (ii) image segmentation [3], (iii) image analysis [4], (iv) image watermarking [5], and (v) content-based image retrieval [6].

The process of color quantization is mainly comprised of two phases: palette design (the selection of a small set of colors that represents the original image colors) and pixel mapping (the assignment of each input pixel to one of the palette colors). The primary objective is to reduce the number of unique colors, N', in an image to K ($K \ll N'$) with minimal distortion. In most applications, 24-bit pixels in the original image are reduced to 8 bits or fewer. Since natural images often contain a large number of colors, faithful representation of these images with a limited size palette is a difficult problem.

Color quantization methods can be broadly classified into two categories [7]: image-independent methods that determine a universal (fixed) palette without regard to any specific image [8], and image-dependent methods that determine a custom (adaptive) palette based on the color distribution of the images. Despite being very fast, image-independent methods usually give poor results since they do not take into account the image contents. Therefore, most of the studies in the literature consider only image-dependent methods, which strive to achieve a better balance between computational efficiency and visual quality of the quantization output.

Numerous image-dependent color quantization methods have been developed in the past three decades. These can be categorized into two families: preclustering methods and postclustering methods [1]. Preclustering methods are mostly based on the statistical analysis of the color distribution of the images. Divisive preclustering methods start with a single cluster that contains all N image pixels. This initial cluster is recursively subdivided until K clusters are obtained. Well-known divisive methods include median-cut [9], octree [10], variance-based method [11], binary splitting [12], greedy orthogonal bipartitioning [13], center-cut [14], and rwm-cut [15]. More recent methods can be found in [16–18]. On the other hand, agglomerative preclustering methods [19–23] start with N singleton clusters each of which contains one image pixel. These clusters are repeatedly merged until K clusters remain. In contrast to preclustering methods that compute the palette only once, postclutering methods first determine an initial palette and then improve it iteratively. Essentially, any data clustering method can be used for this purpose. Since these methods involve iterative or stochastic optimization, they can obtain higher quality results when compared to preclustering methods at the expense of increased computational time. Clustering algorithms adapted to color quantization include k-means [24–27], minmax [28], competitive learning [29–31], fuzzy c-means [32, 33], BIRCH [34], and self-organizing maps [35–37].

In this paper, a fast color quantization method based on the k-means clustering algorithm [38] is presented. The method first reduces the amount of data to be clus-

tered by sampling only the pixels with unique colors. In order to incorporate the color distribution of the pixels into the clustering procedure, each color sample is assigned a weight proportional to its frequency. These weighted samples are then clustered using a fast and exact variant of the k-means algorithm. The set of final cluster centers is taken as the quantization palette.

The rest of the paper is organized as follows. Section 2 describes the conventional k-means clustering algorithm and the proposed modifications. Section 3 describes the experimental setup and presents the comparison of the proposed method with other color quantization methods. Finally, Section 4 gives the conclusions.

2. Color Quantization Using K-Means Clustering Algorithm

The k-means (KM) algorithm is inarguably one of the most widely used methods for data clustering [39]. Given a data set $X = \{\mathbf{x}_1, \ldots, \mathbf{x}_N\} \in \mathbb{R}^D$, the objective of KM is to partition X into K exhaustive and mutually exclusive clusters $S = \{S_1, \ldots, S_k\}$, $\bigcup_{k=1}^{K} S_k = X$, $S_i \cap S_j \equiv \emptyset$ for $i \neq j$ by minimizing the sum of squared error (SSE):

$$\mathrm{SSE} = \sum_{k=1}^{K} \sum_{\mathbf{x}_i \in S_k} \|\mathbf{x}_i - \mathbf{c}_k\|_2^2 \tag{1}$$

where, $\| \, \|_2$ denotes the Euclidean (L_2) norm and $\mathbf{c}_k$ is the center of cluster S_k calculated as the mean of the points that belong to this cluster. This problem is known to be computationally intractable even for $K = 2$ [40], but a heuristic method

developed by Lloyd [41] offers a simple solution. Lloyd's algorithm starts with K arbitrary centers, typically chosen uniformly at random from the data points [42]. Each point is then assigned to the nearest center, and each center is recalculated as the mean of all points assigned to it. These two steps are repeated until a predefined termination criterion is met. The pseudocode for this procedure is given in Algo. (1) (**bold** symbols denote vectors). Here, $m[i]$ denotes the membership of point $\mathbf{x}_i$, i.e. index of the cluster center that is nearest to $\mathbf{x}_i$.

input : $X = \{\mathbf{x}_1, \dots, \mathbf{x}_N\} \in \mathbb{R}^D$ ($N \times D$ input data set)
output: $C = \{\mathbf{c}_1, \dots, \mathbf{c}_K\} \in \mathbb{R}^D$ (K cluster centers)
Select a random subset C of X as the initial set of cluster centers;
while *termination criterion is not met* **do**
  for $(i = 1; i \leq N; i = i + 1)$ **do**
    `Assign` $\mathbf{x}_i$ `to the nearest cluster;`
    $m[i] = \underset{k \in \{1,\dots,K\}}{\operatorname{argmin}} \|\mathbf{x}_i - \mathbf{c}_k\|^2;$
  end
  `Recalculate the cluster centers;`
  for $(k = 1; k \leq K; k = k + 1)$ **do**
    `Cluster` S_k `contains the set of points` $\mathbf{x}_i$ `that are nearest to the center` $\mathbf{c}_k$;
    $S_k = \{\mathbf{x}_i \,|m[i] = k\};$
    `Calculate the new center` $\mathbf{c}_k$ `as the mean of the points that belong to` S_k;
    $\mathbf{c}_k = \frac{1}{|S_k|} \sum_{\mathbf{x}_i \in S_k} \mathbf{x}_i;$
  end
end

Algorithm 1: Conventional K-Means Algorithm

When compared to the preclustering methods, there are two problems with using KM for color quantization. First, due to its iterative nature, the algorithm might require an excessive amount of time to obtain an acceptable output quality. Second, the output is quite sensitive to the initial choice of the cluster centers. In order to

address these problems, we propose several modifications to the conventional KM algorithm:

- **Data sampling**: A straightforward way to speed up KM is to reduce the amount of data, which can be achieved by sampling the original image. Although random sampling can be used for this purpose, there are two problems with this approach. First, random sampling will further destabilize the clustering procedure in the sense that the output will be less predictable. Second, sampling rate will be an additional parameter that will have a significant impact on the output. In order to avoid these drawbacks, we propose a deterministic sampling strategy in which only the pixels with unique colors are sampled. The unique colors in an image can be determined efficiently using a hash table that uses chaining for collision resolution and a universal hash function of the form: $h_a(\mathbf{x}) = \left(\sum_{i=1}^{3} a_i x_i\right) \bmod m$, where $\mathbf{x} = (x_1, x_2, x_3)$ denotes a pixel with red (x_1), green (x_2), and blue (x_3) components, m is a prime number, and the elements of sequence $a = (a_1, a_2, a_3)$ are chosen randomly from the set $\{0, 1, \ldots, m-1\}$.

- **Sample weighting**: An important disadvantage of the proposed sampling strategy is that it disregards the color distribution of the original image. In order to address this problem, each point is assigned a weight that is proportional to its frequency (note that the frequency information is collected during

the data sampling stage). The weights are normalized by the number of pixels in the image to avoid numerical instabilities in the calculations. In addition, Algo. (1) is modified to incorporate the weights in the clustering procedure.

- **Sort-Means algorithm**: The assignment phase of KM involves many redundant distance calculations. In particular, for each point, the distances to each of the K cluster centers are calculated. Consider a point $\mathbf{x}_i$, two cluster centers $\mathbf{c}_a$ and $\mathbf{c}_b$ and a distance metric d, using the triangle inequality, we have $d(\mathbf{c}_a, \mathbf{c}_b) \leq d(\mathbf{x}_i, \mathbf{c}_a) + d(\mathbf{x}_i, \mathbf{c}_b)$. Therefore, if we know that $2d(\mathbf{x}_i, \mathbf{c}_a) \leq d(\mathbf{c}_a, \mathbf{c}_b)$, we can conclude that $d(\mathbf{x}_i, \mathbf{c}_a) \leq d(\mathbf{x}_i, \mathbf{c}_b)$ without having to calculate $d(\mathbf{x}_i, \mathbf{c}_b)$. The compare-means algorithm [43] precalculates the pairwise distances between cluster centers at the beginning of each iteration. When searching for the nearest cluster center for each point, the algorithm often avoids a large number of distance calculations with the help of the triangle inequality test. The sort-means (SM) algorithm [43] further reduces the number of distance calculations by sorting the distance values associated with each cluster center in ascending order. At each iteration, point $\mathbf{x}_i$ is compared against the cluster centers in increasing order of distance from the center $\mathbf{c}_k$ that $\mathbf{x}_i$ was assigned to in the previous iteration. If a center that is far enough from $\mathbf{c}_k$ is reached, all of the remaining centers can be skipped and the procedure continues with the next point. In this way, SM avoids the overhead of going through all the centers. It should

be noted that more elaborate approaches to accelerate KM have been proposed in the literature. These include algorithms based on kd-trees [44], coresets [45], and more sophisticated uses of the triangle inequality [46]. Some of these algorithms [45, 46] are not suitable for low dimensional data sets such as color image data since they incur significant overhead to create and update auxiliary data structures [46]. Others [44] provide computational gains comparable to SM at the expense of significant conceptual and implementation complexity. In contrast, SM is conceptually simple, easy to implement, and incurs very small overhead, which makes it an ideal candidate for color clustering.

We refer to the KM algorithm with the abovementioned modifications as the 'Weighted Sort-Means' (WSM) algorithm. The pseudocode for WSM is given in Algo. (2).

3. Experimental Results and Discussion

3.A. Image set and performance criteria

The proposed method was tested on some of the most commonly used test images in the quantization literature. The natural images in the set included Airplane (512×512, 77,041 (29%) unique colors), Baboon (512×512, 153,171 (58%) unique colors), Boats (787×576, 140,971 (31%) unique colors), Lenna (512×480, 56,164 (23%) unique colors), Parrots (1536×1024, 200,611 (13%) unique colors), and Peppers (512×512, 111,344 (42%) unique colors). The synthetic images included Fish (300×200, 28,170

```
input  : X = {x_1, ..., x_N'} ∈ R^D (N' × D input data set)
         W = {w_1, ..., w_N'} ∈ [0, 1] (N' point weights)
output: C = {c_1, ..., c_K} ∈ R^D (K cluster centers)
Select a random subset C of X as the initial set of cluster centers;
while termination criterion is not met do
    Calculate the pairwise distances between the cluster centers;
    for (i = 1; i ≤ K; i = i + 1) do
        for (j = i + 1; j ≤ K; j = j + 1) do
            d[i][j] = d[j][i] = ||c_i − c_j||^2;
        end
    end
    Construct a K × K matrix M in which row i is a permutation of
    1, ... K that represents the clusters in increasing order of
    distance of their centers from c_i;
    for (i = 1; i ≤ N'; i = i + 1) do
        Let S_p be the cluster that x_i was assigned to in the previous
        iteration;
        p = m[i];
        min_dist = prev_dist = ||x_i − c_p||^2;
        Update the nearest center if necessary;
        for (j = 2; j ≤ K; j = j + 1) do
            t = M[p][j];
            if d[p][t] ≥ 4 prev_dist then
                There can be no other closer center. Stop checking;
                break;
            end
            dist = ||x_i − c_t||^2;
            if dist ≤ min_dist then
                c_t is closer to x_i than c_p;
                min_dist = dist;
                m[i] = t;
            end
        end
    end
    Recalculate the cluster centers;
    for (k = 1; k ≤ K; k = k + 1) do
        Calculate the new center c_k as the weighted mean
        of points that are nearest to it;
        c_k = ( Σ_{m[i]=k} w_i x_i ) / Σ_{m[i]=k} w_i;
    end
end
```

$$\mathbf{c}_k = \left(\sum_{m[i]=k} w_i \mathbf{x}_i \right) \Bigg/ \sum_{m[i]=k} w_i$$

Algorithm 2: Weighted Sort-Means Algorithm

(47%) unique colors) and Poolballs (510×383, 13,604 (7%) unique colors).

The effectiveness of a quantization method was quantified by the Mean Squared Error (MSE) measure:

$$\mathrm{MSE}\left(\mathbf{X}, \hat{\mathbf{X}}\right) = \frac{1}{HW} \sum_{h=1}^{H} \sum_{w=1}^{W} \| \mathbf{x}(h,w) - \hat{\mathbf{x}}(h,w) \|_2^2 \tag{2}$$

where $\mathbf{X}$ and $\hat{\mathbf{X}}$ denote respectively the $H \times W$ original and quantized images in the RGB color space. MSE represents the average distortion with respect to the L_2^2 norm (1) and is the most commonly used evaluation measure in the quantization literature [1, 7]. Note that the Peak Signal-to-Noise Ratio (PSNR) measure can be easily calculated from the MSE value:

$$\mathrm{PSNR} = 20 \log_{10} \left(\frac{255}{\sqrt{\mathrm{MSE}}} \right). \tag{3}$$

The efficiency of a quantization method was measured by CPU time in milliseconds. Note that only the palette generation phase was considered since this is the most time consuming part of the majority of quantization methods. All of the programs were implemented in the C language, compiled with the gcc v4.2.4 compiler, and executed on an Intel®Core™2 Quad Q6700 2.66GHz machine. The time figures were averaged over 100 runs.

3.B. Comparison of WSM against other quantization methods

The WSM algorithm was compared to some of the well-known quantization methods in the literature:

- **Median-cut (MC)** [9]: This method starts by building a $32 \times 32 \times 32$ color histogram that contains the original pixel values reduced to 5 bits per channel by uniform quantization. This histogram volume is then recursively split into smaller boxes until K boxes are obtained. At each step, the box that contains the largest number of pixels is split along the longest axis at the median point, so that the resulting subboxes each contain approximately the same number of pixels. The centroids of the final K boxes are taken as the color palette.

- **Variance-based method (WAN)** [11]: This method is similar to MC, with the exception that at each step the box with the largest weighted variance (squared error) is split along the major (principal) axis at the point that minimizes the marginal squared error.

- **Greedy orthogonal bipartitioning (WU)** [13]: This method is similar to WAN, with the exception that at each step the box with the largest weighted variance is split along the axis that minimizes the sum of the variances on both sides.

- **Neu-quant (NEU)** [35]: This method utilizes a one-dimensional self-organizing map (Kohonen neural network) with 256 neurons. A random subset of N/f pixels is used in the training phase and the final weights of the neurons are taken as the color palette. In the experiments, the highest quality configuration, i.e. $f = 1$, was used.

- **Modified minmax (MMM)** [28]: This method choses the first center $\mathbf{c}_1$ arbitrarily from the data set and the i-th center $\mathbf{c}_i$ ($i = 2, \ldots, K$) is chosen to be the point that has the largest minimum weighted L_2^2 distance (the weights for the red, green, and blue channels are taken as 0.5, 1.0, and 0.25, respectively) to the previously selected centers, i.e. $\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_{i-1}$. Each of these initial centers is then recalculated as the mean of the points assigned to it.

- **Split & Merge (SAM)** [23]: This two-phase method first divides the color space uniformly into B partitions. This initial set of B clusters is represented as an adjacency graph. In the second phase, $(B - K)$ merge operations are performed to obtain the final K clusters. At each step of the second phase, the pair of clusters with the minimum joint quantization error are merged. In the experiments, the initial number of clusters was set to $B = 20K$.

- **Fuzzy c-means (FCM)** [47]: FCM is a generalization of KM in which points can belong to more than one cluster. The algorithm involves the minimization of the functional $J_q(U,V) = \sum_{i=1}^{N} \sum_{k=1}^{K} u_{ik}^q \left\| \mathbf{x}_i - \mathbf{v}_k \right\|_2^2$ with respect to U (a fuzzy K-partition of the data set) and V (a set of prototypes – cluster centers). The parameter q controls the fuzziness of the resulting clusters. At each iteration, the membership matrix U is updated by $u_{ik} = \left(\sum_{j=1}^{K} \left(\left\| \mathbf{x}_i - \mathbf{v}_k \right\|_2 \big/ \left\| \mathbf{x}_i - \mathbf{v}_j \right\|_2 \right)^{2/(q-1)} \right)^{-1}$, which is followed by the update of the prototype matrix V by $\mathbf{v}_k = \left(\sum_{i=1}^{N} u_{ik}^q \mathbf{x}_i \right) \Big/ \left(\sum_{i=1}^{N} u_{ik}^q \right)$. A näive

implementation of the FCM algorithm has a complexity that is quadratic in K. In the experiments, a linear complexity formulation described in [48] was used and the fuzziness parameter was set to $q = 2$ as commonly seen in the fuzzy clustering literature [39].

- **Fuzzy c-means with partition index maximization (PIM)** [32]: This method is an extension of FCM in which the functional to be minimized incorporates a cluster validity measure called the 'partition index' (PI). This index measures how well a point $\mathbf{x}_i$ has been classified and is defined as $P_i = \sum_{k=1}^{K} u_{ik}^q$. The FCM functional can be modified to incorporate PI as follows: $J_q^\alpha(U,V) = \sum_{i=1}^{N}\sum_{k=1}^{K} u_{ik}^q \|\mathbf{x}_i - \mathbf{v}_k\|_2^2 - \alpha\sum_{i=1}^{N} P_i$. The parameter α controls the weight of the second term. The procedure that minimizes $J_q^\alpha(U,V)$ is identical to the one used in FCM except for the membership matrix update equation: $u_{ik} = \left(\sum_{j=1}^{K}\left[(\|\mathbf{x}_i - \mathbf{v}_k\|_2 - \alpha)/(\|\mathbf{x}_i - \mathbf{v}_j\|_2 - \alpha)\right]^{2/(q-1)}\right)^{-1}$. An adaptive method to determine the value of α is to set it to a fraction $0 \le \delta < 0.5$ of the distance between the nearest two centers, i.e. $\alpha = \delta \min\limits_{i \neq j} \|\mathbf{v}_i - \mathbf{v}_j\|_2^2$. Following [32], the fraction value was set to $\delta = 0.4$.

- **Finite-state k-means (FKM)** [25]: This method is a fast approximation for KM. The first iteration is the same as that of KM. In each of the subsequent iterations, the nearest center for a point $\mathbf{x}_i$ is determined from among the K' ($K' \ll K$) nearest neighbors of the center that the point was assigned to in

the previous iteration. When compared to KM, this technique leads to considerable computational savings since the nearest center search is performed in a significantly smaller set of K' centers rather than the entire set of K centers. Following [25], the number of nearest neighbors was set to $K' = 8$.

- **Stable-flags k-means (SKM)** [26]: This method is another fast approximation for KM. The first I' iterations are the same as those of KM. In the subsequent iterations, the clustering procedure is accelerated using the concepts of center stability and point activity. More specifically, if a cluster center $\mathbf{c}_k$ does not move by more than θ units (as measured by the L_2^2 distance) in two successive iterations, this center is classified as stable. Furthermore, points that were previously assigned to the stable centers are classified as inactive. At each iteration, only unstable centers and active points participate in the clustering procedure. Following [26], the algorithm parameters were set to $I' = 10$ and $\theta = 1.0$.

For each KM-based quantization method (except for SKM), two variants were implemented. In the first one, the number of iterations was limited to 10, which makes this variant suitable for time-critical applications. These *fixed-iteration* variants are denoted by the plain acronyms KM, FKM, and WSM. In the second variant, to obtain higher quality results, the method was executed until it converged. Convergence was determined by the following commonly used criterion [38]: $(\text{SSE}_{i-1} - \text{SSE}_i)/\text{SSE}_i \leq \varepsilon$,

where SSE_i denotes the SSE (1) value at the end of the i-th iteration. Following [25,26], the convergence threshold was set to $\varepsilon = 0.0001$. The *convergent* variants of KM, FKM, and WSM are denoted by KM-C, FKM-C, and WSM-C, respectively. Note that since SKM involves at least $I' = 10$ iterations, only the convergent variant was implemented for this method. As for the fuzzy quantization methods, i.e. FCM and PIM, due to their excessive computational requirements, the number of iterations for these methods was limited to 10.

Tables 1-2 compare the performance of the methods at quantization levels $K = \{32, 64, 128, 256\}$ on the test images. Note that, for computational simplicity, random initialization was used in the implementations of FCM, PIM, KM, KM-C, FKM, FKM-C, SKM, WSM, and WSM-C. Therefore, in Table 1, the quantization errors for these methods are specified in the form of mean (μ) and standard deviation (σ) over 100 runs. The best (lowest) error values are shown in **bold**. In addition, with respect to each performance criterion, the methods are ranked based on their mean values over the test images. Table 3 gives the mean ranks of the methods. The last column gives the overall mean ranks with the assumption that each criterion has equal importance. Note that the best possible rank is 1. The following observations are in order:

- ▷ In general, the postclustering methods are more effective but less efficient when compared to the preclustering methods.

- ▷ With respect to distortion minimization, WSM-C outperforms the other methods by a large margin. This method obtains an MSE rank of 1.06, which means that it almost always obtains the lowest distortion.

- ▷ WSM obtains a significantly better MSE rank than its fixed-iteration rivals.

- ▷ Overall, WSM and WSM-C are the best methods.

- ▷ In general, the fastest method is MC, which is followed by SAM, WAN, and WU. The slowest methods are KM-C, FCM, PIM, FKM-C, KM, and SKM.

- ▷ WSM-C is significantly faster than its convergent rivals. In particular, it provides up to 392 times speed up over KM-C with an average of 62.

- ▷ WSM is the fastest post-clustering method. It provides up to 46 times speed up over KM with an average of 14.

- ▷ KM-C, FKM-C, and WSM-C are significantly more stable (particularly when K is small) than their fixed-iteration counterparts as evidenced by their low standard deviation values in Table 1. This was expected since these methods were allowed to run longer which helped them overcome potentially adverse initial conditions.

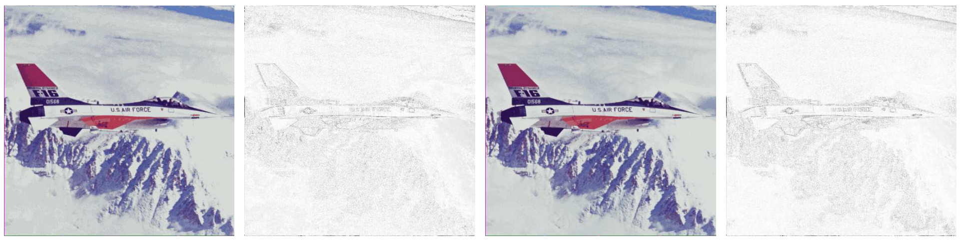
Table 4 gives the mean stability ranks of the methods that involve random initialization. Given a test image and K value combination, the stability of a method is calculated based on the coefficient of variation (σ/μ) as: $100(1 - \sigma/\mu)$, where μ and

σ denote the mean and standard deviation over 100 runs, respectively. Note that the μ and σ values are given in Table 1. Clearly, the higher the stability of a method the better. For example, when $K = 32$, WSM-C obtains a mean MSE of 57.461492 with a standard deviation of 0.861126 on the Airplane image. Therefore, the stability of WSM-C in this case is calculated as $100(1 - 0.861126/57.461492) = 98.50\%$. It can be seen that WSM-C is the most stable method, whereas WSM is the most stable fixed-iteration method.

Figure 1 shows sample quantization results and the corresponding error images. The error image for a particular quantization method was obtained by taking the pixelwise absolute difference between the original and quantized images. In order to obtain a better visualization, pixel values of the error images were multiplied by 4 and then negated. It can be seen that WSM-C and WSM obtain visually pleasing results with less prominent contouring. Furthermore, they achieve the highest color fidelity which is evident by the clean error images that they produce.

Figure 2 illustrates the scaling behavior of WSM with respect to K. It can be seen that the complexity of WSM is sublinear in K, which is due to the intelligent use of the triangle inequality that avoids many distance computations once the cluster centers stabilize after a few iterations. For example, on the Parrots image, increasing K from 2 to 256, results in only about 3.67 fold increase in the computational time (172 ms. vs. 630 ms.).

(a) MMM output (b) MMM error (c) NEU output (d) NEU error



(e) WSM output (f) WSM error (g) WSM-C output (h) WSM-C error

Fig. 1. Sample quantization results for the Airplane image (K=32)

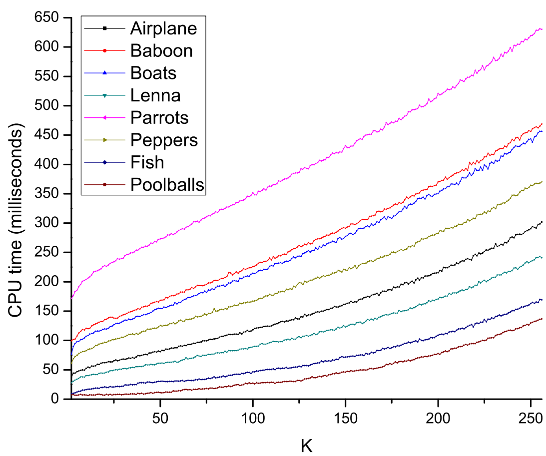


Fig. 2. CPU time for WSM for $K = \{2, \ldots, 256\}$

We should also mention two other KM-based quantization methods [24, 27]. As in the case of FKM and SKM, these methods aim to accelerate KM without degrading its effectiveness. However, they do not address the stability problems of KM and thus provide almost the same results in terms of quality. In contrast, WSM (WSM-C) not only provides considerable speed up over KM (KM-C), but also gives significantly better results especially at lower quantization levels.

4. Conclusions

In this paper, a fast and effective color quantization method called WSM (Weighted Sort-Means) was introduced. The method involves several modifications to the conventional k-means algorithm including data reduction, sample weighting, and the use of triangle inequality to speed up the nearest neighbor search. Two variants of WSM were implemented. Although both have very reasonable computational requirements, the fixed-iteration variant is more appropriate for time-critical applications, while the convergent variant should be preferred in applications where obtaining the highest output quality is of prime importance, or the number of quantization levels or the number of unique colors in the original image is small. Experiments on a diverse set of images demonstrated that the two variants of WSM outperform state-of-the-art quantization methods with respect to distortion minimization. Future work will be directed toward the development of a more effective initialization method for WSM.

The implementation of WSM will be made publicly available as part of the Fourier

image processing and analysis library, which can be downloaded from `http://sourceforge.net/projects/fourier-ipal`.

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Table 1. MSE comparison of the quantization methods

Method	K = 32		K = 64		K = 128		K = 256		K = 32		K = 64		K = 128		K = 256	
	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
	Airplane								Baboon							
MC	124	-	81	-	54	-	41	-	546	-	371	-	248	-	166	-
WAN	117	-	69	-	50	-	39	-	509	-	326	-	216	-	142	-
WU	75	-	47	-	30	-	21	-	422	-	248	-	155	-	99	-
NEU	101	-	47	-	24	-	15	-	363	-	216	-	128	-	84	-
MMM	134	-	82	-	44	-	28	-	489	-	270	-	189	-	120	-
SAM	120	-	65	-	43	-	31	-	396	-	245	-	153	-	99	-
FCM	74	9	44	4	29	2	21	1	415	15	265	10	174	6	119	4
PIM	73	9	45	4	29	2	21	1	413	18	261	13	172	7	117	4
KM	112	25	65	12	36	4	22	2	345	9	206	5	129	2	83	1
KM-C	59	2	35	1	25	0	19	0	329	3	196	1	**123**	1	79	0
FKM	113	19	64	9	36	4	22	1	346	9	206	4	129	2	83	1
FKM-C	59	2	35	1	26	1	19	1	328	3	196	1	**123**	1	79	0
SKM	112	20	63	9	36	4	22	1	343	10	207	6	129	2	83	1
WSM	64	4	36	1	23	1	15	0	345	8	204	3	127	1	81	1
WSM-C	**57**	1	**34**	0	**22**	0	**14**	0	**327**	3	**195**	1	**123**	1	**78**	0
	Boats								Lenna							
MC	200	-	126	-	78	-	57	-	165	-	94	-	71	-	47	-
WAN	198	-	117	-	71	-	45	-	159	-	93	-	61	-	43	-
WU	154	-	87	-	50	-	32	-	130	-	76	-	46	-	29	-
NEU	147	-	79	-	41	-	26	-	119	-	68	-	36	-	23	-
MMM	203	-	114	-	69	-	41	-	139	-	86	-	50	-	34	-
SAM	161	-	95	-	59	-	42	-	135	-	88	-	56	-	40	-
FCM	160	13	99	8	64	5	42	3	132	10	83	7	53	4	38	2
PIM	161	14	99	11	63	5	43	3	136	12	81	6	53	4	38	2
KM	135	11	78	5	47	3	30	1	106	5	61	2	38	1	24	0
KM-C	**115**	1	64	1	39	0	25	0	**97**	1	57	1	35	0	**22**	0
FKM	134	10	77	5	47	3	29	1	107	8	61	2	38	1	24	0
FKM-C	116	1	65	1	39	0	25	0	**97**	1	57	1	35	0	**22**	0
SKM	137	13	77	4	47	2	30	1	107	6	62	2	38	1	24	1
WSM	125	7	68	2	40	1	24	0	103	5	60	2	36	1	23	0
WSM-C	**115**	1	**63**	0	**37**	0	**23**	0	**97**	2	**56**	1	**34**	0	**22**	0
	Parrots								Peppers							
MC	401	-	258	-	144	-	99	-	333	-	213	-	147	-	98	-
WAN	365	-	225	-	146	-	90	-	333	-	215	-	142	-	93	-
WU	291	-	171	-	96	-	59	-	264	-	160	-	101	-	63	-
NEU	306	-	153	-	84	-	47	-	249	-	151	-	83	-	55	-
MMM	332	-	200	-	117	-	73	-	292	-	182	-	113	-	76	-
SAM	276	-	160	-	94	-	60	-	268	-	161	-	100	-	64	-
FCM	297	19	178	14	107	5	69	2	272	15	179	7	120	4	84	3
PIM	295	21	175	12	107	5	69	2	266	14	176	7	119	5	84	3
KM	262	20	149	9	85	4	51	2	232	7	141	4	87	2	54	1
KM-C	237	7	131	3	76	1	46	1	220	2	132	1	**80**	0	51	0
FKM	264	21	150	10	87	4	51	2	231	6	142	4	86	2	55	1
FKM-C	237	7	132	3	77	2	47	1	220	2	132	2	81	1	51	0
SKM	259	16	152	11	86	4	51	2	233	7	142	4	87	2	55	1
WSM	249	13	136	5	79	2	46	1	232	7	139	3	85	1	53	1
WSM-C	**232**	6	**128**	2	**74**	1	**43**	0	**219**	2	**131**	1	**80**	1	**50**	0
	Fish								Poolballs							
MC	276	-	169	-	107	-	68	-	136	-	64	-	38	-	27	-
WAN	311	-	208	-	124	-	77	-	112	-	59	-	45	-	38	-
WU	187	-	111	-	69	-	44	-	68	-	31	-	17	-	11	-
NEU	173	-	107	-	57	-	42	-	104	-	44	-	18	-	9	-
MMM	235	-	136	-	81	-	53	-	166	-	91	-	42	-	20	-
SAM	198	-	120	-	74	-	49	-	91	-	54	-	37	-	20	-
FCM	169	11	110	5	79	3	60	3	153	75	61	30	25	5	14	2
PIM	168	9	111	4	79	3	60	3	149	71	57	26	25	7	14	2
KM	174	24	105	9	64	4	40	2	226	75	129	31	75	17	39	8
KM-C	145	3	90	2	58	2	37	1	94	8	51	5	44	6	29	5
FKM	173	17	105	10	65	4	40	2	229	73	130	44	78	15	37	6
FKM-C	144	3	90	2	59	2	38	1	95	9	55	10	45	8	27	5
SKM	177	19	105	9	65	4	40	2	167	35	120	15	71	13	37	7
WSM	148	3	91	3	55	1	33	0	69	10	31	6	14	2	**7**	0
WSM-C	**142**	4	**85**	1	**52**	1	**32**	0	**62**	6	**27**	3	**13**	1	**7**	0

Table 2. CPU time comparison of the quantization methods

Method	K = 32	K = 64	K = 128	K = 256	K = 32	K = 64	K = 128	K = 256
	Airplane				Baboon			
MC	10	10	**11**	**12**	10	**10**	**11**	**13**
WAN	13	14	15	18	14	15	16	20
WU	16	16	16	16	16	15	16	17
NEU	70	142	265	514	67	134	254	485
MMM	123	206	367	696	126	207	375	702
SAM	**7**	**8**	13	25	**9**	20	56	112
FCM	2739	5285	10612	21079	2737	5285	10612	21081
PIM	2410	5038	10402	20913	2488	5091	10407	20846
KM	584	1005	1791	3314	592	1012	1800	3317
KM-C	17688	43850	74814	71908	3136	7070	13164	25657
FKM	189	222	299	505	189	223	299	508
FKM-C	4111	6144	6057	5376	746	934	1171	1959
SKM	530	903	1593	2952	547	927	1610	2961
WSM	68	92	145	301	147	188	270	477
WSM-C	257	359	522	1180	401	565	814	1580
	Boats				Lenna			
MC	19	**18**	**19**	**21**	9	8	10	**10**
WAN	24	24	26	29	12	15	15	17
WU	28	26	28	28	15	15	14	15
NEU	122	232	453	853	61	123	244	465
MMM	219	367	656	1237	116	193	346	654
SAM	**17**	19	21	32	**8**	**7**	**9**	13
FCM	4695	9141	18350	36471	2545	4954	9953	19770
PIM	4075	8555	17784	36071	2348	4820	9832	19681
KM	986	1727	3087	5729	536	939	1673	3101
KM-C	9853	22622	53858	111047	3457	6698	11927	23762
FKM	326	385	509	804	170	205	281	478
FKM-C	2393	3158	4007	6056	788	878	1167	1886
SKM	908	1551	2756	5105	485	837	1493	2778
WSM	136	174	255	464	52	68	110	244
WSM-C	486	614	853	1647	149	212	329	883
	Parrots				Peppers			
MC	**57**	**58**	**59**	**61**	10	**10**	**11**	**12**
WAN	81	82	83	86	13	14	16	18
WU	86	87	86	87	16	17	17	17
NEU	476	849	1571	2914	70	135	262	493
MMM	758	1265	2282	4286	125	206	371	700
SAM	74	77	103	150	**8**	11	29	53
FCM	16096	31734	63871	126554	2739	5288	10624	21107
PIM	14620	30159	61891	124794	2499	5107	10425	20883
KM	3309	5918	10657	19828	564	996	1785	3309
KM-C	23949	61168	119907	242439	3387	7761	14839	31893
FKM	1100	1302	1698	2519	181	219	295	500
FKM-C	5464	8557	9529	10482	869	1017	1262	2233
SKM	3072	5429	9506	17599	523	905	1605	2971
WSM	250	298	399	639	107	138	201	373
WSM-C	634	820	1261	2149	327	466	648	1387
	Fish				Poolballs			
MC	6	**5**	**7**	**6**	**9**	**9**	**9**	**11**
WAN	5	6	8	12	10	10	12	14
WU	8	9	8	9	12	13	12	13
NEU	12	27	58	110	51	103	192	353
MMM	23	34	59	112	87	145	263	498
SAM	**4**	6	9	17	**9**	10	16	23
FCM	610	1209	2428	4832	1999	3940	7913	15719
PIM	560	1171	2401	4806	1586	3406	6817	13257
KM	128	229	404	757	396	703	1281	2400
KM-C	1147	2777	4395	5233	3339	13294	14912	22637
FKM	39	49	78	187	133	158	213	369
FKM-C	267	346	420	893	913	1565	1285	2036
SKM	121	207	361	672	380	653	1173	2174
WSM	25	32	57	173	**9**	15	34	136
WSM-C	85	109	182	572	24	34	94	356

Table 3. Performance rank comparison of the quantization methods

Method	MSE rank	Time rank	Mean rank
MC	13.97	**1.38**	7.67
WAN	13.66	2.84	8.25
WU	8.47	3.31	5.89
NEU	6.31	6.00	6.16
MMM	12.31	7.63	9.97
SAM	10.09	2.53	6.31
FCM	10.31	13.94	12.13
PIM	9.81	12.94	11.38
KM	7.56	11.34	9.45
KM-C	3.03	15.00	9.02
FKM	7.91	7.75	7.83
FKM-C	3.88	11.53	7.70
SKM	8.06	10.25	9.16
WSM	3.56	5.28	**4.42**
WSM-C	**1.06**	8.25	4.66

Table 4. Stability rank comparison of the quantization methods

Method	MSE rank
FCM	9.36
PIM	9.56
KM	8.31
KM-C	2.84
FKM	8.10
FKM-C	3.41
SKM	7.11
WSM	3.92
WSM-C	**2.02**